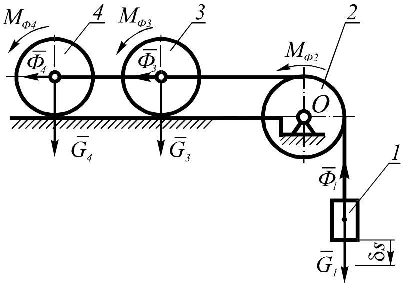
Problem D-1 2020

To determine the acceleration of the body 1 at rolling without slipping there can be used the general equation of dynamics:

$$G_1\delta S - \Phi_1\delta S - M_{\Phi 2}\frac{\delta S}{r_2} - M_{\Phi 3}\frac{\delta S}{r_3} - \Phi_3\delta S - M_{\Phi 4}\frac{\delta S}{r_3} - \Phi_4\delta S = 0\ .$$



Moreover, $\Phi_1 = \Phi_3 = \Phi_4 = m_1 a_1$; $\varepsilon_3 = \dfrac{a_1}{r_3}$; $\varepsilon_4 = \dfrac{a_1}{r_4}$;

$$M_{\Phi 2} = \frac{m\,r_2^2}{2}\cdot\frac{a_1}{r_2};\quad M_{\Phi 3} = \frac{m\,r_3^2}{2}\cdot\frac{a_1}{r_3};\quad M_{\Phi 4} = \frac{m\,r_4^2}{2}\cdot\frac{a_1}{r_4}\ .$$

Substitution in the dynamics general equation gives:

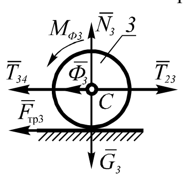
$$m_1 g - m_1 a_1 - \frac{m a_1}{2} - \frac{m a_1}{2} - m a_1 - m a_1 - m a_1 = 0\ .$$

From where
$$a_1 = \frac{m_1 g}{m_1 + 4m}; \qquad (1)$$

The rolling is without slipping, so $F_{\text{тр}} \le f N$. The normal reactions of the supporting surface are $N_3 = N_4 = mg$.

To determine the friction force it is necessary to compose the equation of torques relative to the center of mass of the roller:

$$\sum M_{iC3} = 0;\ M_{\Phi 3} - F_{\text{тр}3} r_3 = 0\ .$$



Than
$$F_{\text{тр}3} = \frac{m\,r_3^2}{2}\cdot\frac{a_1}{r_3^2} = \frac{m a_1}{2}\ .$$

Similarly it is defined that $F_{\text{тр}4} = ma_1$.

So, the third roller starts slipping at $a_1 > 2fg$, athe forth roller – at $a_1 > fg$. I. e. the forth roller slips at a less mass m_1. Its value can be found from the equation (1):

$$\frac{m_1 g}{m_1 + 4m} = fg\ ; \quad m_1 = \frac{4mf}{1-f}.$$

At a full slipping we have:

$$\begin{cases} m_4 a_1 = T_{34} - fmg; \\ m_3 a_1 = T_{23} - T_{34} - fmg; \\ J_{20}\dfrac{a_1}{r} = (T_1 - T_{23})r; \\ m_1 a_1 = G_1 - T_1 r. \end{cases}$$

A sequential substitution of the expressions for forces in the last equation of the system gives:

$$m_1 a_1 = m_1 g - \frac{m_1 a_1}{2} - m_3 a_1 - fmg - m_4 a_1 - fmg\ .$$

From where $$a_1 = \frac{m_1 g - 2fmg}{m_1 + 2{,}5m}\ .$$

For the start of the roller 3 slipping the following condition must be satisfied

$$a_1 = 2fg\ .$$

Than

$$\frac{m_1 g - 2fmg}{m_1 + 2{,}5m} = 2fg\ ; \quad m_1 = \frac{7fm}{1-2f}.$$

If $f \geq 0{,}5$, than the value of mass $m_1 < 0$. This fact means that the roller 3 will never move with slipping.

So,

$$\text{If } f < 0{,}5, \quad \text{then} \quad \frac{4mf}{1-f} < m_1 < \frac{7fm}{1-2f};$$

$$\text{if } f \geq 0{,}5, \quad \text{then} \quad \frac{4mf}{1-f} < m_1 < \infty.$$