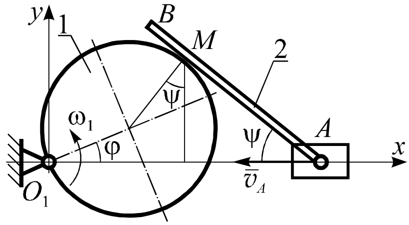
Problem K-1 2020



The coordinates of the point M for an arbitrary position in the reference system with origin at the point O_1 are:

$x_M = R\cos\varphi + R\sin\psi = O_1A - AM\cos\psi = O_1A_0 - v_A t - AM\cos\psi$;

$y_M = R\sin\varphi + R\cos\psi = AM\sin\psi$.

Than the projections of the point M velocity vector are

$$v_{M_x} = -R\sin\varphi\frac{d\varphi}{dt} + R\cos\psi\frac{d\psi}{dt} = -v_A - \frac{dAM}{dt}\cos\psi + AM\sin\psi\frac{d\psi}{dt},$$

$$v_{M_y} = R\cos\varphi\frac{d\varphi}{dt} - R\sin\psi\frac{d\psi}{dt} = \frac{dAM}{dt}\sin\psi + AM\cos\psi\frac{d\psi}{dt}.$$

As $\dfrac{d\varphi}{dt} = \omega_1$, $\dfrac{d\psi}{dt} = \omega_2$ и $\dfrac{dAM}{dt} = v_{M2}$, than

$$\begin{aligned} -R\sin\varphi\cdot\omega_1 + R\cos\psi\cdot\omega_2 &= -v_A - v_{M2}\cos\psi + AM\sin\psi\cdot\omega_2, \\ R\cos\varphi\cdot\omega_1 - R\sin\psi\cdot\omega_2 &= v_{M2}\sin\psi + AM\cos\psi\cdot\omega_2. \end{aligned} \qquad (1)$$

In the considered position $\varphi = 0$, $\psi = \alpha = 30°$, $AM = R\sqrt{3}$, so

$$\begin{aligned} R\cos 30^\circ\cdot\omega_2 &= -v_A - v_{M2}\cos 30^\circ + R\sqrt{3}\sin 30^\circ\cdot\omega_2; \\ \omega_1 R - R\sin 30^\circ\omega_2 &= v_{M2}\sin 30^\circ + R\sqrt{3}\cos 30^\circ\cdot\omega_2. \end{aligned} \qquad (2)$$

From the equation (1) it is obtained

$$\omega_2 R\frac{\sqrt{3}}{2} = -v_A - v_{M2}\frac{\sqrt{3}}{2} + R\frac{\sqrt{3}}{2}\omega_2; \quad v_{M2} = -v_A\frac{2}{\sqrt{3}} = -3\cdot\frac{2}{\sqrt{3}} = -2\sqrt{3}\,\text{cm/s}$$

Than the equation (2) will be

$$\omega_1 R - \omega_2\frac{R}{2} = -2\sqrt{3}\cdot\frac{1}{2} + \frac{3}{2}\omega_2 R.$$

From it $$\omega_2 = \frac{\omega_1 R + \sqrt{3}}{2R} = \frac{2\cdot 4\cdot\sqrt{3} + \sqrt{3}}{2\cdot 4\cdot\sqrt{3}} = \frac{9}{8}\ rad/s.$$

$$v_{M1} = (\omega_1 + \omega_2)R = (2 + \frac{9}{8})\cdot 4\sqrt{3} = \frac{25\sqrt{3}}{2}\ \text{cm/s}.$$

After the differentiation of the system of equations (1) with respect to time

$$-\omega_1 R\cos\varphi\frac{d\varphi}{dt}+\frac{d\omega_2}{dt}R\cos\psi-\omega_2 R\sin\psi\frac{d\psi}{dt}=-\frac{dv_{M2}}{dt}\cos\psi+v_{M2}\sin\psi\frac{d\psi}{dt}+$$
$$+\frac{dAM}{dt}\sin\psi\cdot\omega_2+AM\cos\psi\frac{d\psi}{dt}\cdot\omega_2+AM\sin\psi\frac{d\omega_2}{dt};$$
$$-\omega_1 R\sin\varphi\frac{d\varphi}{dt}-\frac{d\omega_2}{dt}R\sin\psi-\omega_2 R\cos\psi\frac{d\psi}{dt}=\frac{dv_{M2}}{dt}\sin\psi+v_{M2}\cos\psi\frac{d\psi}{dt}+$$
$$+\frac{dAM}{dt}\cos\psi\cdot\omega_2-AM\sin\psi\frac{d\psi}{dt}\cdot\omega_2+AM\cos\psi\frac{d\omega_2}{dt}.$$

For the considered position

$$-\omega_1^2 R+\varepsilon_2 R\cos 30^\circ-\omega_2^2 R\sin 30^\circ=$$
$$=-a_{M2}^\tau\cos 30^\circ+2v_{M2}\omega_2\sin 30°+\omega_2^2 AM\cos 30^\circ+\varepsilon_2 AM\sin 30^\circ;$$
$$-\varepsilon_2 R\sin 30^\circ-\omega_2^2 R\cos 30^\circ=$$
$$=a_{M2}^\tau\sin 30^\circ+2v_{M2}\omega_2\cos 30°-\omega_2^2 AM\sin 30^\circ+\varepsilon_2 AM\cos 30^\circ.$$

Now there is a need to solve the system of equations with respect to a_{M2}^τ and ε_2.

$$-R\omega_1^2+\varepsilon_2 R\frac{\sqrt{3}}{2}-\omega_2^2 R\frac{1}{2}=-a_{M2}^\tau\frac{\sqrt{3}}{2}+2v_{M2}\omega_2\cdot\frac{1}{2}+R\sqrt{3}\cdot\frac{\sqrt{3}}{2}\cdot\omega_2^2+R\sqrt{3}\cdot\frac{1}{2}\varepsilon_2;$$
$$-\varepsilon_2 R\frac{1}{2}-\omega_2^2 R\frac{\sqrt{3}}{2}=a_{M2}^\tau\frac{1}{2}+2v_{M2}\omega_2\cdot\frac{\sqrt{3}}{2}-R\sqrt{3}\cdot\frac{1}{2}\omega_2^2+R\sqrt{3}\cdot\frac{\sqrt{3}}{2}\cdot\varepsilon_2.$$

From where

$$a_{M2}^\tau=\frac{2\omega_2^2 R+\omega_1^2 R+v_{M2}\omega_2}{\frac{\sqrt{3}}{2}}=\frac{2\cdot\left(\frac{9}{8}\right)^2\cdot 4\sqrt{3}+2^2\cdot 4\sqrt{3}-2\sqrt{3}\cdot\frac{9}{8}}{\frac{\sqrt{3}}{2}}=\frac{191}{4}\frac{\text{cm}}{\text{s}^2};$$

$$\varepsilon_2=-\frac{a_{M2}^\tau\cdot\frac{1}{2}+v_{M2}\omega_2\sqrt{3}}{2R}=-\frac{\frac{191}{4}\cdot\frac{1}{2}-2\sqrt{3}\cdot\frac{9}{8}\cdot\sqrt{3}}{2R}=-\frac{137}{16R}\ \text{rad/s}^2.$$

Than the relative acceleration of the point M in its movement relative to the cam

$$a_{M1}^\tau=\varepsilon_2 R=-\frac{137}{16}\ \text{cm/s}^2.$$

$$a_{M1}^n=(\omega_1+\omega_2)^2 R=\left(2+\frac{9}{8}\right)^2 4\sqrt{3}\ \text{cm/s}^2.$$

$$a_{M1}=\sqrt{\left(\frac{137}{16}\right)^2+\left(\frac{625\cdot\sqrt{3}}{16}\right)^2}=\frac{\sqrt{137^2+3\cdot 625^2}}{16}=68{,}2\ \text{cm/s}^2.$$