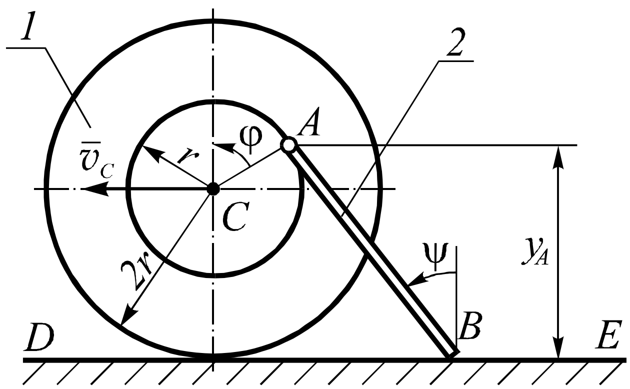
Problem K-2 2020



When the point A is at its upper position the rod is in the upright position. The angular velocity of the rod can be determined using the method of coordinates:

$$y_A = 2r + r\cos\varphi = AB\cos\psi; \qquad (1)$$

$$\dot{y}_A = -r\sin\varphi\cdot\dot{\varphi} = -AB\sin\psi\cdot\dot{\psi};$$

$$\dot{\psi} = \frac{r\sin\varphi\cdot\dot{\varphi}}{AB\sin\psi} = \frac{r\sin\varphi\cdot\dot{\varphi}}{3r\sin\psi} = \frac{\sin\varphi}{3\sin\psi}\cdot\dot{\varphi}.$$

Taking into account that $AB = 3\,r$, and the formula (1), it follows that

$$\sin\psi = \sqrt{1-\cos^2\psi} = \sqrt{1-\left(\frac{2+\cos\varphi}{3}\right)^2} = \frac{\sqrt{5-4\cos\varphi-\cos^2\varphi}}{3};$$

$$\dot{\psi} = \frac{\sin\varphi\cdot\dot{\varphi}}{\sqrt{5-4\cos\varphi-\cos^2\varphi}}.$$

In the considered position $\varphi = 0$, the obtained equation is an uncertainty of the 0/0 type. So, to find the value of the angular velocity ω_2, it is necessary to define the function limit

$$\omega_2 = \lim_{\varphi\to 0}\frac{\sin\varphi}{\sqrt{5-4\cos\varphi-\cos^2\varphi}}\omega_1;$$

$$\omega_2^2 = \lim_{\varphi\to 0}\frac{\sin^2\varphi\cdot\omega_1^2}{5-4\cos\varphi-\cos^2\varphi} = \lim_{\varphi\to 0}\frac{2\sin\varphi\cdot\cos\varphi}{4\sin\varphi+2\cos\varphi\sin\varphi}\omega_1^2 = \lim_{\varphi\to 0}\frac{2\cos\varphi}{4+2\cos\varphi}\omega_1^2 = \frac{\omega_1^2}{3}.$$

Therefore,
$$\omega_2 = \frac{\omega_1}{\sqrt{3}} = \frac{v_C}{2r\sqrt{3}}.$$