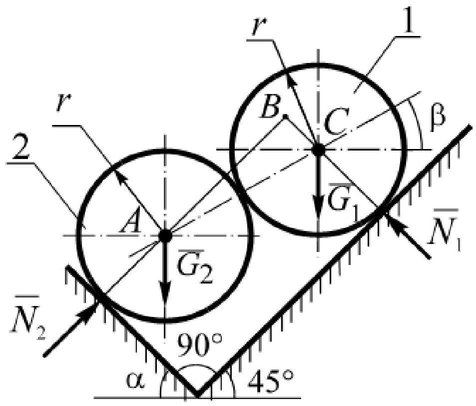
Problem S-1 2020



1. The case of smooth surfaces.

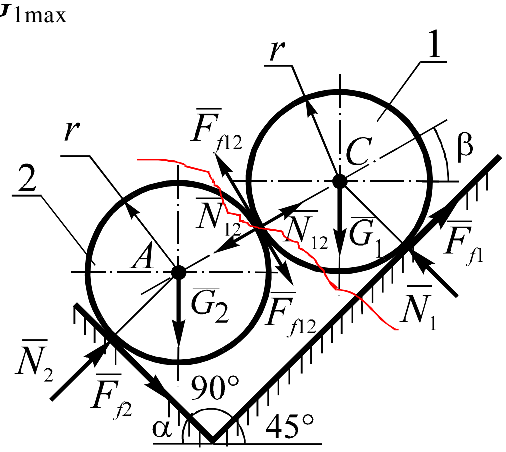
$$\sum M_{iB}=0;\ G_2 AB\cos 45° - G_1 BC\cos 45° = 0$$

$$AB = AC\cos 15°;\ BC = AC\sin 15°$$

$$G_2 AC\cos 15° - G_1 AC\sin 15° = 0$$

$$G_1 = G_2\cot 15° = \left(2+\sqrt{3}\right)G_2$$

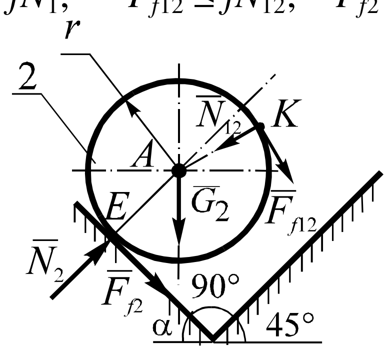
2. a) Determining $G_{1\max}$



$$\sum M_{iC}=0;\quad F_{f1}\cdot r - F_{f12}\cdot r = 0 \quad\Rightarrow\quad F_{f1} = F_{f12}$$

$$\sum M_{iA}=0;\quad F_{f2}\cdot r - F_{f12}\cdot r = 0 \quad\Rightarrow\quad F_{f2} = F_{f12}$$

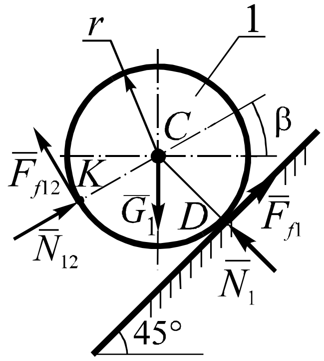
$$F_{f1} \le fN_1;\qquad F_{f12} \le fN_{12};\quad F_{f2} \le fN_2$$



$$\sum M_{iE}=0;\quad -G_2 r\sin 45^\circ + N_{12} r\sin 15^\circ + F_{f12}(r + r\cos 15^\circ) = 0 \qquad (1)$$

$$\sum M_{iK}=0;\quad G_2 r\cos 30^\circ - N_2 r\sin 15^\circ + F_{f2}(r + r\cos 15^\circ) = 0$$

Substituting $F_{f12} = fN_{12}$ and $F_{f2} = fN_2$ it is obtained that $N_2 > N_{12}$. Therefore, taking into account that $F_{f2} = F_{f12}$, it can be concluded that the slipping can not start at the point E.



$$\sum M_{iD}=0; \quad G_1 r\sin 45^\circ - N_{12} r\cos 15^\circ - F_{f12}(r + r\sin 15^\circ)=0 \tag{2}$$

$$\sum M_{iK}=0; \quad -G_1 r\cos 60^\circ + N_1 r\cos 15^\circ + F_{f1}(r + r\sin 15^\circ)=0 \tag{3}$$

Substituting $F_{f12}=fN_{12}$ and $F_{f1}=fN_1$ it is obtained that $N_{12}>N_1$. So, considering that $F_{f1} = F_{f12}$, it can be concluded that the slipping can't start at the point K. In other words, the imbalance can be caused by the start of slipping at the point D.

Thus, $F_{f2} < fN_2$; $F_{f12} < fN_{12}$; $F_{f1} = fN_1$.

After solving the equations (1) and (2) in common, it is obtained

$$G_1 \frac{\sqrt{2}}{2} - N_{12}\cos 15° - F_{f12}(1+\sin 15°) = 0$$

$$-G_2 \frac{\sqrt{2}}{2} + N_{12}\sin 15° + F_{f12}(1+\cos 15°) = 0$$

$$N_{12} = \frac{G_1 \frac{\sqrt{2}}{2} - F_{f12}(1+\sin 15°)}{\cos 15°}$$

$$-G_2 \frac{\sqrt{2}}{2} + G_1 \frac{\sqrt{2}}{2}\tan 15° - F_{f12}(1+\sin 15°)\tan 15° + F_{f12}(1+\cos 15°)\tan 15° = 0$$

$$F_{f12} = \frac{G_1 \frac{\sqrt{2}}{2}\tan 15° - G_2 \frac{\sqrt{2}}{2}}{(1+\cos 15°) - (1+\sin 15°)\tan 15°}.$$

From the other side, from the equation (3)

$$-G_1 \cos 60^\circ + \frac{F_{f1}}{f}\cos 15^\circ + F_{f1}(1+\sin 15^\circ) = 0;$$

$$-G_1 \cos 60^\circ + \frac{F_{f1}}{f}\cos 15^\circ + F_{f1}(1+\sin 15^\circ) = 0$$

$$F_{f1} = \frac{fG_1 \cos 60°}{\cos 15° + f(1+\sin 15^\circ)}$$

Taking into account that $F_{f1} = F_{f12}$, we have

$$\frac{fG_1\cos 60°}{\cos 15° + f\,(1+\sin 15^\circ)} = \frac{G_1\dfrac{\sqrt{2}}{2}\tan 15° - G_2\dfrac{\sqrt{2}}{2}}{(1+\cos 15°)-(1+\sin 15°)\tan 15°};$$

$$fG_1\left((1+\cos 15°)-(1+\sin 15°)\tan 15°\right) = \left(G_1\sqrt{2}\tan 15° - G_2\sqrt{2}\right)\left(\cos 15° + f\,(1+\sin 15^\circ)\right);$$

$$G_2\sqrt{2}\left(\cos 15° + f\,(1+\sin 15^\circ)\right) =$$

$$= G_1\sqrt{2}\tan 15°\left(\cos 15° + f\,(1+\sin 15^\circ)\right) - fG_1\left((1+\cos 15°)-(1+\sin 15°)\tan 15°\right).$$

Finally it is obtained

$$G_{1\max} = \frac{G_2\sqrt{2}\left(\cos 15° + f\,(1+\sin 15^\circ)\right)}{\sqrt{2}\tan 15°\left(\cos 15° + f\,(1+\sin 15^\circ)\right) - f\left((1+\cos 15°)-(1+\sin 15°)\tan 15°\right)}.$$

б) Finding $G_{1\min}$

In this case the directions of the friction forces should be changed to the opposite ones.

Conducting the similar calculations, we can show that at the beginning of slipping at the point *D* the sign in front of *f* in the final expression for the $G_{1\max}$ should be also changed to the opposite one

$$G_{1\min} = \frac{G_2\sqrt{2}\left(\cos 15° - f\,(1+\sin 15^\circ)\right)}{\sqrt{2}\tan 15°\left(\cos 15° - f\,(1+\sin 15^\circ)\right) + f\left((1+\cos 15°)-(1+\sin 15°)\tan 15°\right)}.$$

The answer for the problem is a range from the $G_{1\min}$ to the $G_{1\max}$.

It should be noted that the obtained equations are valid only for the small values of the friction coefficients *f*. If the values of *f* are big it should be additionally analyzed the possible start of slipping at the point *E* when determining $G_{1\min}$, and it is also necessary to check the possibility of the cylinders jamming.