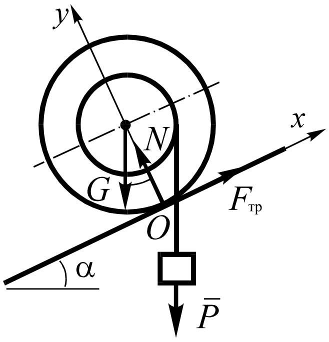
S1-2021



$\sum M_{iO}=0$: $GR\sin\alpha-P(r-R\sin\alpha)=0$

$r-R\sin\alpha>0$

$\sin\alpha<\frac{r}{R}$

$$\begin{cases}\sum F_{iy}=0 & -G\cos\alpha+N-P\sin\alpha=0\\ \sum F_{ix}=0 & F_{тр}-(P+G)\sin\alpha=0\end{cases}$$

$N=(P+G)\cos\alpha$

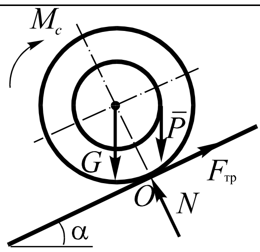
$F_{тр}=(P+G)\sin\alpha$

$F_{тр}\le fN$

$(P+G)\sin\alpha\le f(P+G)\cos\alpha$

$\tan\alpha\le f$

$$\boxed{\alpha<\arcsin\frac{r}{R};\ \ \alpha\le\arctan f}$$



$M_{c\max}=N\cdot\delta=(P+G)\cos\alpha\cdot\delta=0$

$GR\sin\alpha-P_{\min}(r-R\sin\alpha)-(P_{\min}+G)\cos\alpha\cdot\delta=0$

$$P_{\min}=\frac{G(R\sin\alpha-\delta\cos\alpha)}{r-R\sin\alpha+\delta\cos\alpha};$$

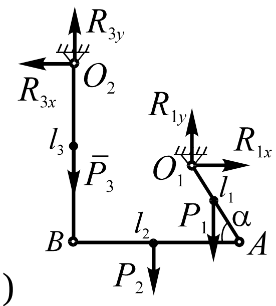
if $\delta\cos\alpha>R\sin\alpha$, then $P_{\min}=0$

$GR\sin\alpha-P_{\max}(r-R\sin\alpha)+(P_{\max}+G)\delta\cos\alpha)$

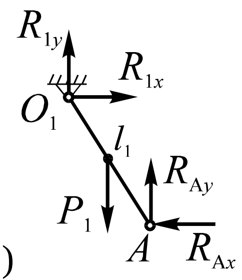
$$P_{\max}=\frac{G(R\sin\alpha+\delta\cos\alpha)}{r-R\sin\alpha-\delta\cos\alpha};$$

if $\delta\cos\alpha\ge r-R\sin\alpha$, then $P_{\max}=\infty$.

S2-2021



(1)



(2)

$$(1)\ \sum M_{O2}=0;\ -P_2\frac{l_2}{2}-P_1(l_2-\frac{l_1}{2}\cos\alpha)+R_{1x}(l_3-l_1\sin\alpha)+R_{1y}(l_2-l_1\cos\alpha)=0;$$

$$(2)\ \sum M_{iA}=0;\ P_1\frac{l_1}{2}\cos\alpha-R_{1y}l_1\cos\alpha-R_{1x}l_1\sin\alpha=0;$$

$$\frac{P_1}{2}-R_{1y}-R_{1x}\cdot\tan\alpha=0;\ R_{1y}=\frac{P_1}{2}-R_{1x}\cdot\tan\alpha=0;$$

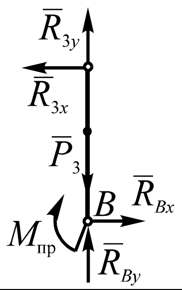
$$-P_2\frac{l_2}{2}-P_1(l_2-\frac{l_1}{2}\cos\alpha)+R_{1x}(l_3-l_1\sin\alpha)+(\frac{P_1}{2}-R_{1x}\tan\alpha)(l_2-l_1\cos\alpha)=0;$$

$$R_{1x}(l_3-l_1\sin\alpha-l_2\tan\alpha+l_1\sin\alpha)=P_2\frac{l_2}{2}+P_1(l_2-\frac{l_1}{2}\cos\alpha)-P_1\frac{l_2}{2}+\frac{P_1l_1\cos\alpha}{2};$$

$$R_{1x}(l_3-l_2tg\alpha)=P_2\frac{l_2}{2}+P_1\frac{l_2}{2};\ R_{1x}=\frac{(P_2+P_1)\frac{l_2}{2}}{l_3-l_2\tan\alpha}=R_{3x};$$

$$R_{1y}=\frac{P_1}{2}-R_{1x}\tan\alpha=\frac{P_1}{2}-\frac{(P_2+P_1)\frac{l_2}{2}\tan\alpha}{l_3-l_2\tan\alpha};$$

$$R_{3y}=P_1+P_2+P_3-R_{1y}=\frac{P_2}{2}+P_2+P_3+\frac{(P_2+P_1)\frac{l_2}{2}\tan\alpha}{l_3-l_2\tan\alpha};$$

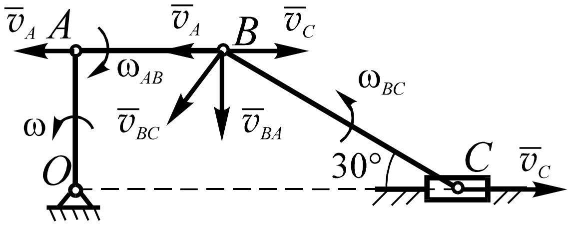


$$\boxed{R_{Bx}=R_{3x}=\frac{(P_2+P_1)\frac{l_2}{2}}{l_3-l_2\tan\alpha}};$$

$$-M_{\text{пр}}+R_{3x}\cdot l_3=0;\ \boxed{M_{\text{пр}}=R_{3x}\cdot l_3=\frac{(P_2+P_1)l_2l_3}{2(l_3-l_2\tan\alpha)}};$$

$$R_{3y}-P_3+R_{By}=0;\ \boxed{R_{By}=P_3-R_{3y}=-\frac{P_1}{2}-P_2-\frac{(P_2+P_1)l_2\tan\alpha}{2(l_3-l_2\tan\alpha)}}$$

K1–2021



$$\vec{V}_B = \vec{V}_A + \vec{V}_{BA} = \vec{V}_C + \vec{V}_{BC}$$

$$V_{Bx} = -V_A = V_C - V_{BC}\sin 30^\circ;$$

$$V_{By} = -V_{BA} = V_{BC}\cos 30^\circ;$$

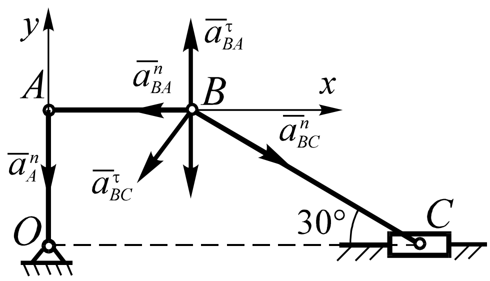
$$V_{BC} = \frac{V_A + V_C}{\sin 30^\circ} = 4V_A;$$

$$V_{BA} = V_{BC}\cos 30^\circ = 2V_A\sqrt{3}$$

$$\omega_{BA} = \frac{V_{BA}}{AB} = \frac{2V_A\sqrt{3}}{l};\ \ \omega_{BC} = \frac{V_{BC}}{BC} = \frac{4V_A}{2l} = \frac{2V_A}{l};\ \ \boxed{\frac{\omega_{AB}}{\omega_{BC}} = \frac{2V_A\sqrt{3}}{2V_A} = \sqrt{3};}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}^{\,n}_{BA} + \vec{a}^{\,\tau}_{BA} = \vec{a}_C + \vec{a}^{\,n}_{BC} + \vec{a}^{\,\tau}_{BC};$$

$$a^n_{BA} = \omega^2_{BA}\cdot AB = \frac{12V_A^2}{l};\ \ a^n_{BC} = \omega^2_{BC}\cdot BC = \frac{8V_A^2}{l}.$$



$$a_{Bx} = -a^n_{BA} = a^n_{BC}\cos 30^\circ - a^\tau_{BC}\sin 30^\circ;$$

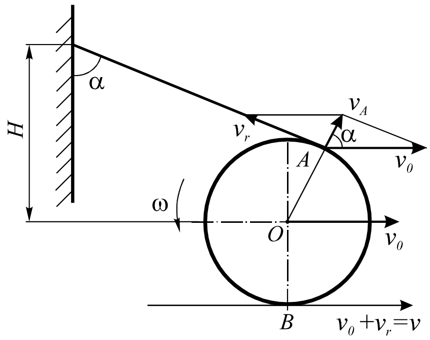
$$a_{By} = -a_A + a^\tau_{BA} = -a^n_{BC}\sin 30^\circ - a^\tau_{BC}\cos 30^\circ$$

$$a^\tau_{BC} = (a^n_{BA} + a^n_{BC}\cos 30^\circ)/\sin 30^\circ = \frac{8V_A^2}{l}(\sqrt{3}+3),$$

$$a^\tau_{BA} = a^n_A - a^n_{BC}\sin 30^\circ - a^\tau_{BC}\cos 30^\circ = -\frac{3V_A^2}{l}(4\sqrt{3}+5)$$

$$\boxed{\frac{|\varepsilon_{AB}|}{|\varepsilon_{BC}|} = \frac{|a^\tau_{BA}|}{AB}\cdot\frac{BC}{|a^\tau_{BC}|} = \frac{3}{4}\cdot\frac{4\sqrt{3}+5}{\sqrt{3}+3} = \frac{7\sqrt{3}+3}{8} = 1.89}$$

K2-2021



Suppose, for example, that at some time moment the cylinder axis has velocity equal to v_0 and the angular velocity of cylinder is ω. At cylinder's movement the thread is always starched, so the velocity of cylinder point A (the point touches the thread) is perpendicular to the thread.

$$\bar{v}_A=\bar{v}_0+\bar{v}_r; \quad v_r = \omega r;$$

$$v_0 \sin\alpha = \omega r.$$

The point B of the cylinder touches the plane and its velocity is,

$$v_B = v_0+\omega r = v; \; v_0 + v_0 \sin\alpha = v$$

$$\boxed{v_0=\frac{v}{1+\sin\alpha};}$$

$$\boxed{\omega_{\text{thread}}=\frac{v_0\cos\alpha}{l}=\frac{v\cos\alpha}{(1+\sin\alpha)l};}$$

$$H=l\cos\alpha+r\sin\alpha=\text{const}, \; l=\frac{H-r\sin\alpha}{\cos\alpha}.$$

$$\boxed{a_0=\frac{dv_0}{dt}=-\frac{v\cos\alpha\,\omega_{\text{thread}}}{(1+\sin\alpha)^2}=-\frac{v^2\cos^2\alpha}{l(1+\sin\alpha)^2};}$$

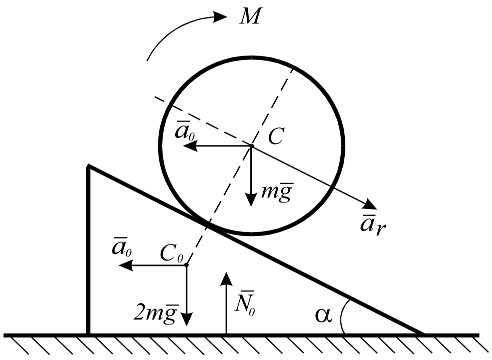
$$\varepsilon_{\text{thread}}=\frac{d\omega_{\text{thread}}}{dt}=\frac{v}{l^2}\,\frac{-\sin\alpha\cdot\omega_{\text{thread}}(1+\sin\alpha)l-\cos\alpha\cdot\left(\cos\alpha\,\omega_{\text{thread}}l+(1+\sin\alpha)\dfrac{dl}{dt}\right)}{(1+\sin\alpha)^2}=;$$

$$=-\frac{v}{l^2}\,\frac{\omega_{\text{thread}}l+\dfrac{dl}{dt}}{1+\sin\alpha}=$$

$$v=v_0+\omega_{\text{thread}}r+\frac{dl}{dt}; \; \frac{dl}{dt}=\omega_{\text{thread}}r(l\tan\alpha-r).$$

$$\boxed{\varepsilon_{\text{thread}}=-\frac{v^2\cos\alpha(l+\sin\alpha-r\cos\alpha)}{l^3(1+\sin\alpha)^2}.}$$

D1-2021

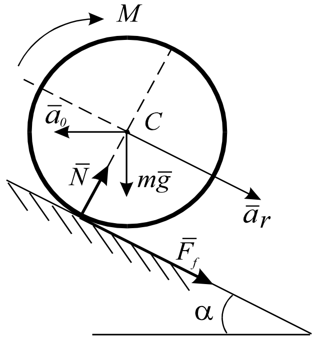


The theorem on the motion of the center of mass of the system

$$2ma_0 + m(a_0 - a_r\cos\alpha) = 0$$

$$ma_r\sin\alpha = 3mg - N_0$$

$$a_r = 3\frac{a_0}{\cos\alpha},\ N_0 = 3mg + ma_r\sin\alpha = 3mg\left(1 + \frac{a_0}{g}\mathrm{tg}\alpha\right)$$



$$N - mg\cos\alpha = -ma_0\sin\alpha;$$

$$mg\sin\alpha + F_f = m(a_r - a_0\cos\alpha);$$

$$N = mg\cos\alpha - ma_0\sin\alpha;\ F_f = -mg\sin\alpha + ma_0\left(\frac{3}{\cos\alpha} - \cos\alpha\right).$$

$$\frac{1}{2}mr^2\varepsilon = M - F_f r;$$

1. Case for no slipping.

$$\varepsilon^s = \frac{a_r}{r} = \frac{3a_0}{r\cos\alpha};$$

$$F_f = \frac{M}{r} - \frac{3ma_0}{2\cos\alpha};$$

$$\frac{M}{r} - \frac{3ma_0}{2\cos\alpha} = -mg\sin\alpha + ma_0\left(\frac{3}{\cos\alpha} - \cos\alpha\right);$$

$$a_0 = \frac{M + mgr\sin\alpha}{mr\left(\frac{4,5}{\cos\alpha} - \cos\alpha\right)};$$

$$N_0^s = 3mg\left(1 - \frac{M + mgr\sin\alpha}{mgr(4,5 - \cos^2\alpha)}\sin\alpha\right) = \frac{3(3.5mgr - M\sin\alpha)}{r(4.5 - \cos^2\alpha)};$$

2. Slipping at big values of torques

It should be defined the minimal value of M_1 torque when the slipping starts

$$F_f = fN;\ \frac{M_1}{r} - \frac{3ma_0}{2\cos\alpha} = f(mg\cos\alpha - ma_0\sin\alpha)$$

$$M_1 = mgr\left[\frac{a_0}{g}\left(\frac{3}{2\cos\alpha} - f\sin\alpha\right) + f\cos\alpha\right]$$

$$f(mg\cos\alpha - ma_0\sin\alpha) = -mg\sin\alpha + ma_0\left(\frac{3}{\cos\alpha} - \cos\alpha\right)$$

$$g\sin\alpha + fg\cos\alpha = a_0\left(\frac{3}{\cos\alpha} - \cos\alpha + f\sin\alpha\right)$$

$$a_0 = g\frac{\sin\alpha + f\cos\alpha}{3 - \cos^2\alpha + f\sin\alpha\cdot\cos\alpha}\cos\alpha$$

$$M_1 = mgr\left[\frac{(3 - f\sin 2\alpha)(\sin\alpha + f\cos\alpha)}{2(3 - \cos^2\alpha + f\sin\alpha\cos\alpha)} + f\cos\alpha\right]$$

$$N_0 = 3mg\left(1 - \frac{\sin\alpha - f\cos\alpha}{3 - \cos^2\alpha - f\sin\alpha\cos\alpha}\sin\alpha\right) = \frac{6mg}{3 - \cos^2\alpha - f\sin\alpha\cos\alpha}$$

Slipping at small values of torques

$$F_f = -fN,\ M = M_2$$

$$M_2 = mgr\left[\frac{(3 + f\sin 2\alpha)(\sin\alpha - f\cos\alpha)}{2(3 - \cos^2\alpha - f\sin\alpha\cdot\cos\alpha)} - f\cos\alpha\right]$$

$$N_0 = 3mg\left(1 + \frac{\sin\alpha - f\cos\alpha}{3 - \cos^2\alpha - f\sin\alpha\cdot\cos\alpha}\sin\alpha\right)$$

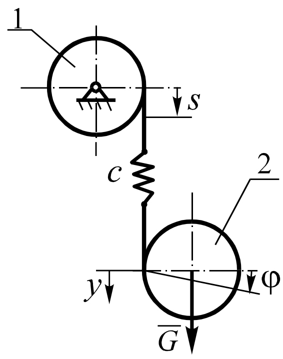
Finally we get

If $0 < M < M_2$, then $N_0 = \dfrac{6mg}{3 - \cos^2\alpha - f\sin\alpha\cos\alpha}$;

if $M_2 < M < M_1$, then $N_0 = N_0^s = \dfrac{3(3.5mgr - M\sin\alpha)}{r(4.5 - \cos^2\alpha)}$;

if $M > M_1$, then $N_0 = \dfrac{6mg}{3 - \cos^2\alpha + f\sin\alpha\cos\alpha}$.

D2-2021



$$T=\frac{mr^2}{2\cdot 2}\cdot\frac{\dot{s}^2}{r^2}+\frac{m(\dot{y}+r\dot{\phi})^2}{2}+\frac{mr^2}{2\cdot 2}\dot{\phi}^2=\frac{m\cdot\dot{s}^2}{4}+\frac{m(\dot{y}+r\dot{\phi})^2}{2}+\frac{mr^2}{4}\dot{\phi}^2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{s}}\right)=\frac{m\ddot{s}}{2};\quad \frac{d}{dt}\left(\frac{\partial T}{\partial\dot{y}}\right)=m(\ddot{y}+r\ddot{\varphi});\quad \frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\varphi}}\right)=m(\ddot{y}+r\ddot{\varphi})r+\frac{mr^2\ddot{\varphi}}{2};$$

$$Q_s=c(y-s);\; Q_y=mg-c(y-s);\; Q_\varphi=mgr.$$

$$\begin{cases}\frac{m\ddot{s}}{2}=c(y-s);\\ m(\ddot{y}+r\ddot{\varphi})=mg-c(y-s);\\ \ddot{y}+\frac{3}{2}r\ddot{\varphi}=g.\end{cases}$$

$$r\ddot{\varphi}=\frac{2}{3}g-\frac{2}{3}\ddot{y};$$

$$m(\frac{\ddot{y}}{3}+\frac{2}{3}g)=mg-c(y-s);$$

$$\frac{m\ddot{y}}{3}=\frac{mg}{3}-c(y-s);$$

$$\ddot{y}=g-3\frac{c}{m}(y-s);$$

$$\frac{\ddot{s}}{2}=\frac{c}{m}(y-s);\quad y=\frac{m\ddot{s}}{2c}+s;$$

$$\ddot{y}=g-3\frac{\ddot{s}}{2};$$

$$\frac{s^{\mathrm{IV}}}{2}=\frac{c}{m}(\ddot{y}-\ddot{s});$$

$$\frac{ms^{\mathrm{IV}}}{2c}+\ddot{s}=g-\frac{3\ddot{s}}{2};$$

$$\frac{ms^{\mathrm{IV}}}{2c}+\frac{5}{2}\ddot{s}=g;$$

$$\frac{m}{2c}\lambda^4+\frac{5}{2}\lambda^2=0;$$

$$\left(\frac{m}{2c}\lambda^2+\frac{5}{2}\right)\lambda^2=0;$$

$$\lambda_1=\lambda_2=0;\ \lambda_{3,4}=\pm\sqrt{\frac{5c}{m}}i;$$

$$s=C_1+C_2t+C_3\sin\sqrt{\frac{5c}{m}}t+C_4\cos\sqrt{\frac{5c}{m}}t+\frac{gt^2}{5};$$

$$\ddot{s}=g\frac{2}{5}-\frac{5c}{m}\left(C_3\sin\sqrt{\frac{5c}{m}}t+C_4\cos\sqrt{\frac{5c}{m}}t\right).$$

At $t = 0$, $s = 0$, $\dot{s}=0$, $\ddot{s}=0$; $\dddot{s}=0$.

$$s(0) = C_1+C_4 =0;\ \dot{s}(0)=C_2+C_3\sqrt{\frac{5c}{m}}=0;$$

$$\ddot{s}(0)=-C_4\frac{5c}{m}+g\frac{2}{5}=0;\ \dddot{s}(0)=C_3\left(\frac{5c}{m}\right)^{3/2}=0.$$

$$C_2=C_3=0;\ C_4=\frac{2mg}{25c};\ C_1=-\frac{2mg}{25c}.$$

$$\ddot{s}=\frac{2g}{5}-\frac{5c}{m}\left(\frac{2mg}{25c}\cos\sqrt{\frac{5c}{m}}t\right)=\frac{2g}{5}\left(1-\cos\sqrt{\frac{5c}{m}}t\right).$$

$$\ddot{s}_{max}=4g/5;\ \boxed{\ddot{\varphi}_{1max}=\frac{\ddot{s}_{max}}{r}=\frac{4g}{5r}}\ \text{at}\ \cos\sqrt{\frac{5c}{m}}t=-1.$$

$$\sqrt{\frac{5c}{m}}t=\pi;\ \boxed{t=\pi\sqrt{\frac{m}{5c}}.}$$