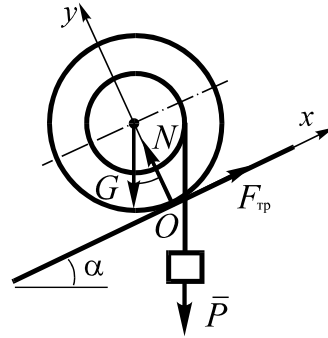


S1-2021



$$\sum M_{iO} = 0: GR\sin\alpha - P(r - R\sin\alpha) = 0$$

$$r - R\sin\alpha > 0$$

$$\sin\alpha < \frac{r}{R}$$

$$\sum F_{iy} = 0 \quad \left\{ \begin{array}{l} -G\cos\alpha + N - P\sin\alpha = 0 \\ \sum F_{ix} = 0 \quad \left\{ \begin{array}{l} F_{tp} - (P+G)\sin\alpha = 0 \\ N = (P+G)\cos\alpha \\ F_{tp} = (P+G)\sin\alpha \\ F_{tp} \leq fN \\ (P+G)\sin\alpha \leq f(P+G)\cos\alpha \\ \tan\alpha \leq f \end{array} \right. \end{array} \right.$$

$$\sum F_{ix} = 0 \quad \left\{ \begin{array}{l} F_{tp} - (P+G)\sin\alpha = 0 \\ N = (P+G)\cos\alpha \\ F_{tp} = (P+G)\sin\alpha \\ F_{tp} \leq fN \\ (P+G)\sin\alpha \leq f(P+G)\cos\alpha \\ \tan\alpha \leq f \end{array} \right.$$

$$N = (P+G)\cos\alpha$$

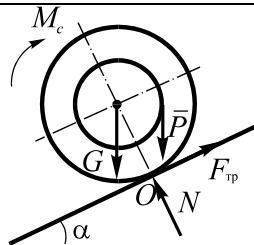
$$F_{tp} = (P+G)\sin\alpha$$

$$F_{tp} \leq fN$$

$$(P+G)\sin\alpha \leq f(P+G)\cos\alpha$$

$$\tan\alpha \leq f$$

$$\alpha < \arcsin \frac{r}{R}; \quad \alpha \leq \arctan f$$



$$M_{c\max} = N \cdot \delta = (P+G)\cos\alpha \cdot \delta = 0$$

$$GR\sin\alpha - P_{\min}(r - R\sin\alpha) - (P_{\min} + G)\cos\alpha \cdot \delta = 0$$

$$P_{\min} = \frac{G(R\sin\alpha - \delta\cos\alpha)}{r - R\sin\alpha + \delta\cos\alpha};$$

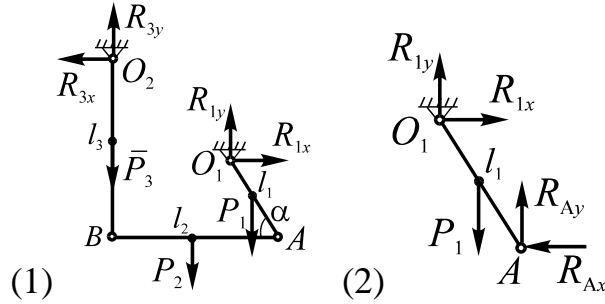
$$\text{if } \delta\cos\alpha > R\sin\alpha, \text{ then } P_{\min} = 0$$

$$GR\sin\alpha - P_{\max}(r - R\sin\alpha) + (P_{\max} + G)\delta\cos\alpha = 0$$

$$P_{\max} = \frac{G(R\sin\alpha + \delta\cos\alpha)}{r - R\sin\alpha - \delta\cos\alpha};$$

$$\text{if } \delta\cos\alpha \geq r - R\sin\alpha, \text{ then } P_{\max} = \infty.$$

S2-2021



$$(1) \sum M_{O_2} = 0; -P_2 \frac{l_2}{2} - P_1 \left(l_2 - \frac{l_1}{2} \cos \alpha \right) + R_{1x} (l_3 - l_1 \sin \alpha) + R_{1y} (l_2 - l_1 \cos \alpha) = 0;$$

$$(2) \sum M_{iA} = 0; P_1 \frac{l_1}{2} \cos \alpha - R_{1y} l_1 \cos \alpha - R_{1x} l_1 \sin \alpha = 0;$$

$$\frac{P_1}{2} - R_{1y} - R_{1x} \cdot \tan \alpha = 0; \Rightarrow R_{1y} = \frac{P_1}{2} - R_{1x} \cdot \tan \alpha = 0;$$

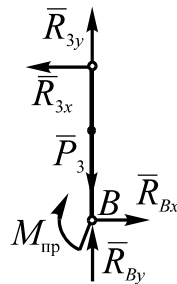
$$-P_2 \frac{l_2}{2} - P_1 \left(l_2 - \frac{l_1}{2} \cos \alpha \right) + R_{1x} (l_3 - l_1 \sin \alpha) + \left(\frac{P_1}{2} - R_{1x} \tan \alpha \right) (l_2 - l_1 \cos \alpha) = 0;$$

$$R_{1x} (l_3 - l_1 \sin \alpha - l_2 \tan \alpha + l_1 \sin \alpha) = P_2 \frac{l_2}{2} + P_1 \left(l_2 - \frac{l_1}{2} \cos \alpha \right) - P_1 \frac{l_2}{2} + \frac{P_1 l_1 \cos \alpha}{2};$$

$$R_{1x} (l_3 - l_2 \tan \alpha) = P_2 \frac{l_2}{2} + P_1 \frac{l_2}{2}; R_{1x} = \frac{(P_2 + P_1) l_2}{2(l_3 - l_2 \tan \alpha)} = R_{3x};$$

$$R_{1y} = \frac{P_1}{2} - R_{1x} \tan \alpha = \frac{P_1}{2} - \frac{(P_2 + P_1) l_2}{2(l_3 - l_2 \tan \alpha)} \tan \alpha;$$

$$R_{3y} = P_1 + P_2 + P_3 - R_{1y} = \frac{P_2}{2} + P_2 + P_3 + \frac{(P_2 + P_1) \frac{l_2}{2} \tan \alpha}{l_3 - l_2 \tan \alpha};$$

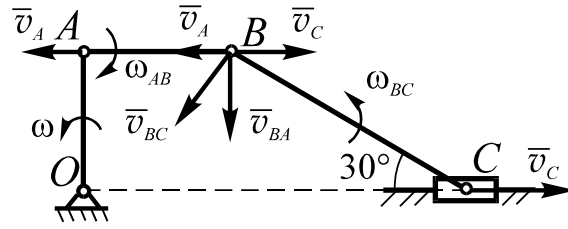


$$R_{Bx} = R_{3x} = \frac{(P_2 + P_1) \frac{l_2}{2}}{l_3 - l_2 \tan \alpha};$$

$$-M_{np} + R_{3x} \cdot l_3 = 0; M_{np} = R_{3x} \cdot l_3 = \frac{(P_2 + P_1) l_2 l_3}{2(l_3 - l_2 \tan \alpha)};$$

$$R_{3y} - P_3 + R_{By} = 0; R_{By} = P_3 - R_{3y} = -\frac{P_1}{2} - P_2 - \frac{(P_2 + P_1) l_2 \tan \alpha}{2(l_3 - l_2 \tan \alpha)}$$

K1-2021



$$\vec{V}_B = \vec{V}_A + \vec{V}_{BA} = \vec{V}_C + \vec{V}_{BC}$$

$$V_{Bx} = -V_A = V_C - V_{BC} \sin 30^\circ;$$

$$V_{By} = -V_{BA} = -V_{BC} \cos 30^\circ;$$

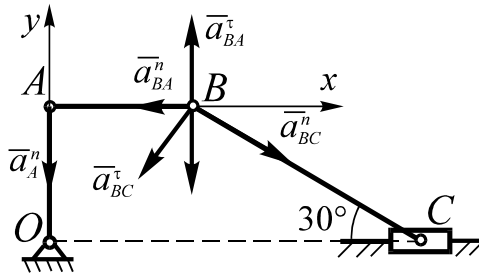
$$V_{BC} = \frac{V_A + V_C}{\sin 30^\circ} = 4V_A;$$

$$V_{BA} = V_{BC} \cos 30^\circ = 2V_A \sqrt{3}$$

$$\omega_{BA} = \frac{V_{BA}}{AB} = \frac{2V_A \sqrt{3}}{l}; \quad \omega_{BC} = \frac{V_{BC}}{BC} = \frac{4V_A}{2l} = \frac{2V_A}{l}; \quad \boxed{\frac{\omega_{AB}}{\omega_{BC}} = \frac{2V_A \sqrt{3}}{2V_A} = \sqrt{3};}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^n + \vec{a}_{BA}^\tau = \vec{a}_C + \vec{a}_{BC}^n + \vec{a}_{BC}^\tau;$$

$$a_{BA}^n = \omega_{BA}^2 \cdot AB = \frac{12V_A^2}{l}; \quad a_{BC}^n = \omega_{BC}^2 \cdot BC = \frac{8V_A^2}{l}.$$



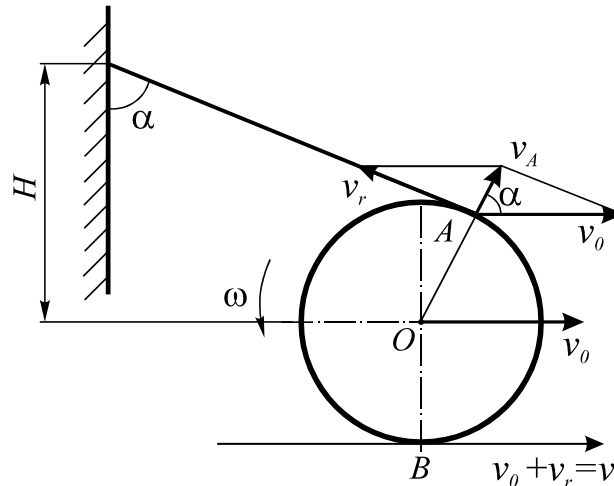
$$a_{Bx} = -a_{BA}^n = a_{BC}^n \cos 30^\circ - a_{BC}^\tau \sin 30^\circ;$$

$$a_{By} = -a_A + a_{BA}^\tau = -a_{BC}^n \sin 30^\circ - a_{BC}^\tau \cos 30^\circ$$

$$a_{BC}^\tau = (a_{BA}^n + a_{BC}^n \cos 30^\circ) / \sin 30^\circ = \frac{8V_A^2}{l} (\sqrt{3} + 3),$$

$$a_{BA}^\tau = a_A^n - a_{BC}^n \sin 30^\circ - a_{BC}^\tau \cos 30^\circ = -\frac{3V_A^2}{l} (4\sqrt{3} + 5)$$

$$\boxed{\frac{|\varepsilon_{AB}|}{|\varepsilon_{BC}|} = \frac{|a_{BA}^\tau|}{AB} \cdot \frac{BC}{|a_{BC}^\tau|} = \frac{3}{4} \cdot \frac{4\sqrt{3} + 5}{\sqrt{3} + 3} = \frac{7\sqrt{3} + 3}{8} = 1.89}$$



Пусть в некоторый момент времени ось цилиндра имеет скорость v_0 , а угловая скорость цилиндра равна ω . Нить при движении цилиндра всегда натянута, поэтому скорость точки A цилиндра, касающейся нити, перпендикулярна нити.

$$\vec{v}_A = \vec{v}_0 + \vec{v}_r; \quad v_r = \omega r;$$

$$v_0 \sin \alpha = \omega r.$$

Скорость точки B цилиндра, касающейся подставки,

$$v_B = v_0 + \omega r = v; \quad v_0 + v_0 \sin \alpha = v$$

$$v_0 = \frac{v}{1 + \sin \alpha};$$

$$\omega_{\text{нити}} = \frac{v_0 \cos \alpha}{l} = \frac{v \cos \alpha}{(1 + \sin \alpha) l};$$

$$H = l \cos \alpha + r \sin \alpha = \text{const}, \quad l = \frac{H - r \sin \alpha}{\cos \alpha}.$$

$$a_0 = \frac{dv_0}{dt} = -\frac{v \cos \alpha \omega_{\text{нити}}}{(1 + \sin \alpha)^2} = -\frac{v^2 \cos^2 \alpha}{l(1 + \sin \alpha)^2};$$

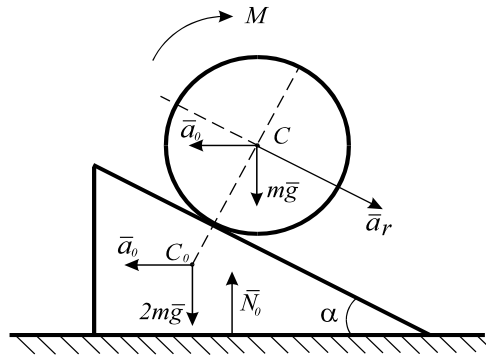
$$\varepsilon_{\text{нити}} = \frac{d\omega_{\text{нити}}}{dt} = \frac{v}{l^2} \frac{-\sin \alpha \cdot \omega_{\text{нити}} (1 + \sin \alpha) l - \cos \alpha \cdot \left(\cos \alpha \omega_{\text{нити}} l + (1 + \sin \alpha) \frac{dl}{dt} \right)}{(1 + \sin \alpha)^2} =$$

$$= -\frac{v}{l^2} \frac{\omega_{\text{нити}} l + \frac{dl}{dt}}{1 + \sin \alpha} =$$

$$v = v_0 + \omega_{\text{нити}} r + \frac{dl}{dt}; \quad \frac{dl}{dt} = \omega_{\text{нити}} r (l \tan \alpha - r).$$

$$\varepsilon_{\text{нити}} = -\frac{v^2 \cos \alpha (l + \sin \alpha - r \cos \alpha)}{l^3 (1 + \sin \alpha)^2}.$$

D1-2021

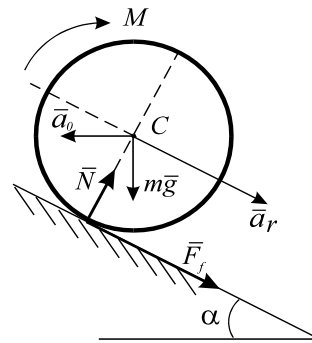


Теорема о движении центра масс системы

$$2ma_0 + m(a_0 - a_r \cos \alpha) = 0$$

$$ma_r \sin \alpha = 3mg - N_0$$

$$a_r = 3 \frac{a_0}{\cos \alpha}, \quad N_0 = 3mg - ma_r \sin \alpha = 3mg \left(1 - \frac{a_0}{g} \operatorname{tg} \alpha \right)$$



$$N - mg \cos \alpha = -ma_0 \sin \alpha;$$

$$mg \sin \alpha + F_f = m(a_r - a_0 \cos \alpha);$$

$$N = mg \cos \alpha - ma_0 \sin \alpha; \quad F_f = -mg \sin \alpha + ma_0 \left(\frac{3}{\cos \alpha} - \cos \alpha \right).$$

$$\frac{1}{2} mr^2 \varepsilon = M - F_f r;$$

1. Случай без проскальзывания

$$\varepsilon^s = \frac{a_r}{r} = \frac{3a_0}{r \cos \alpha};$$

$$F_f = \frac{M}{r} - \frac{3ma_0}{2 \cos \alpha};$$

$$\frac{M}{r} - \frac{3ma_0}{2 \cos \alpha} = -mg \sin \alpha + ma_0 \left(\frac{3}{\cos \alpha} - \cos \alpha \right);$$

$$a_0 = \frac{M + mgr \sin \alpha}{mr \left(\frac{4,5}{\cos \alpha} - \cos \alpha \right)};$$

$$N_0^s = 3mg \left(1 - \frac{M + mgr \sin \alpha}{mgr(4,5 - \cos^2 \alpha)} \sin \alpha \right) = \frac{3(3,5mgr - M \sin \alpha)}{r(4,5 - \cos^2 \alpha)};$$

2. Проскальзывание при больших моментах

Определяем минимальное значение M_1 момента, при котором начинается скольжение

$$F_f = fN; \quad \frac{M_1}{r} - \frac{3ma_0}{2\cos\alpha} = f(mg\cos\alpha - ma_0\sin\alpha)$$

$$M_1 = mgr \left[\frac{a_0}{g} \left(\frac{3}{2\cos\alpha} - f\sin\alpha \right) + f\cos\alpha \right]$$

$$f(mg\cos\alpha - ma_0\sin\alpha) = -mg\sin\alpha + ma_0 \left(\frac{3}{\cos\alpha} - \cos\alpha \right)$$

$$g\sin\alpha + fg\cos\alpha = a_0 \left(\frac{3}{\cos\alpha} - \cos\alpha + f\sin\alpha \right)$$

$$a_0 = g \frac{\sin\alpha + f\cos\alpha}{3 - \cos^2\alpha + f\sin\alpha \cdot \cos\alpha} \cos\alpha$$

$$M_1 = mgr \left[\frac{(3 - f\sin 2\alpha)(\sin\alpha + f\cos\alpha)}{2(3 - \cos^2\alpha + f\sin\alpha \cos\alpha)} + f\cos\alpha \right]$$

$$N_0 = 3mg \left(1 - \frac{\sin\alpha + f\cos\alpha}{3 - \cos^2\alpha + f\sin\alpha \cdot \cos\alpha} \sin\alpha \right) = \frac{6mg}{3 - \cos^2\alpha + f\sin\alpha \cos\alpha}$$

Проскальзывание при малых моментах

$$F_f = -fN, \quad M = M_2$$

$$M_2 = mgr \left[\frac{(3 + f\sin 2\alpha)(\sin\alpha - f\cos\alpha)}{2(3 - \cos^2\alpha - f\sin\alpha \cdot \cos\alpha)} - f\cos\alpha \right]$$

$$N_0 = 3mg \left(1 - \frac{\sin\alpha - f\cos\alpha}{3 - \cos^2\alpha - f\sin\alpha \cos\alpha} \sin\alpha \right) = \frac{6mg}{3 - \cos^2\alpha - f\sin\alpha \cos\alpha}$$

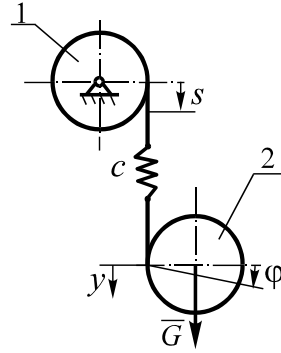
Окончательно получаем

$$\text{Если } 0 < M < M_2, \text{ то } N_0 = \frac{6mg}{3 - \cos^2\alpha - f\sin\alpha \cos\alpha};$$

$$\text{если } M_2 < M < M_1, \text{ то } N_0 = N_0^s = \frac{3(3.5mgr - M\sin\alpha)}{r(4.5 - \cos^2\alpha)};$$

$$\text{если } M > M_1, \text{ то } N_0 = \frac{6mg}{3 - \cos^2\alpha + f\sin\alpha \cos\alpha}.$$

D2-2021



$$T = \frac{mr^2}{2 \cdot 2} \cdot \frac{\dot{s}^2}{r^2} + \frac{m(\dot{y} + r\dot{\phi})^2}{2} + \frac{mr^2}{2 \cdot 2} \dot{\phi}^2 = \frac{m \cdot \dot{s}^2}{4} + \frac{m(\dot{y} + r\dot{\phi})^2}{2} + \frac{mr^2}{4} \dot{\phi}^2$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{s}} \right) = \frac{m\ddot{s}}{2}; \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{y}} \right) = m(\ddot{y} + r\ddot{\phi}); \quad \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\phi}} \right) = m(\ddot{y} + r\ddot{\phi})r + \frac{mr^2}{2} \ddot{\phi};$$

$$Q_s = c(y - s); \quad Q_y = mg - c(y - s); \quad Q_\phi = mgr.$$

$$\begin{cases} \frac{m\ddot{s}}{2} = c(y - s); \\ m(\ddot{y} + r\ddot{\phi}) = mg - c(y - s); \\ \ddot{y} + \frac{3}{2}r\ddot{\phi} = g. \end{cases}$$

$$r\ddot{\phi} = \frac{2}{3}g - \frac{2}{3}\ddot{y};$$

$$m\left(\frac{\ddot{y}}{3} + \frac{2}{3}g\right) = mg - c(y - s);$$

$$\frac{m\ddot{y}}{3} = \frac{mg}{3} - c(y - s);$$

$$\ddot{y} = g - 3\frac{c}{m}(y - s);$$

$$\frac{\ddot{s}}{2} = \frac{c}{m}(y - s); \quad y = \frac{m\ddot{s}}{2c} + s;$$

$$\ddot{y} = g - 3\frac{\ddot{s}}{2};$$

$$\frac{s^{IV}}{2} = \frac{c}{m}(\ddot{y} - \ddot{s});$$

$$\frac{ms^{IV}}{2c} + \ddot{s} = g - \frac{3\ddot{s}}{2};$$

$$\frac{ms^{IV}}{2c} + \frac{5}{2}\ddot{s} = g;$$

$$\frac{m}{2c}\lambda^4 + \frac{5}{2}\lambda^2 = 0;$$

$$\left(\frac{m}{2c}\lambda^2 + \frac{5}{2}\right)\lambda^2 = 0;$$

$$\lambda_1 = \lambda_2 = 0; \lambda_{3,4} = \pm \sqrt{\frac{5c}{m}}i;$$

$$s = C_1 + C_2 t + C_3 \sin \sqrt{\frac{5c}{m}} t + C_4 \cos \sqrt{\frac{5c}{m}} t + \frac{gt^2}{5};$$

$$\ddot{s} = g \frac{2}{5} - \frac{5c}{m} \left(C_3 \sin \sqrt{\frac{5c}{m}} t + C_4 \cos \sqrt{\frac{5c}{m}} t \right).$$

При $t = 0, s = 0, \dot{s} = 0, \ddot{s} = 0; \ddot{\ddot{s}} = 0$.

$$s(0) = C_1 + C_4 = 0; \dot{s}(0) = C_2 + C_3 \sqrt{\frac{5c}{m}} = 0;$$

$$\ddot{s}(0) = -C_4 \frac{5c}{m} + g \frac{2}{5} = 0; \ddot{\ddot{s}}(0) = C_3 \left(\frac{5c}{m}\right)^{3/2} = 0.$$

$$C_2 = C_3 = 0; C_4 = \frac{2mg}{25c}; C_1 = -\frac{2mg}{25c}.$$

$$\ddot{s} = \frac{2g}{5} - \frac{5c}{m} \left(\frac{2mg}{25c} \cos \sqrt{\frac{5c}{m}} t \right) = \frac{2g}{5} \left(1 - \cos \sqrt{\frac{5c}{m}} t \right).$$

$$\ddot{s}_{\max} = 4g/5; \ddot{\phi}_{1\max} = \frac{\ddot{s}_{\max}}{r} = \frac{4g}{5r} \text{ при } \cos \sqrt{\frac{5c}{m}} t = -1.$$

$$\sqrt{\frac{5c}{m}} t = \pi; t = \pi \sqrt{\frac{m}{5c}}.$$